Slides Week 39

Some properties of Statistics

$X_1,\ldots,X_n$ are $N\left(\mu,\sigma^2\right)$

$\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i$ and $S^2=\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2$ are independent

$\bar{X}\sim N\left(\mu,\frac{\sigma^2}{n}\right)$, $\frac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1)$

T-statistic: $\frac{\bar{X}-\mu}{\frac{S}{\sqrt{n}}}$, In general $T_p=\frac{N(0,1)}{\sqrt{\frac{\chi^2(p)}{p}}}$

$$Var\left[T_p\right]=\frac{p}{p-2}$$

$$F_{p,q} \text{ statistic} = \frac{\frac{\chi^2(p)}{p}}{\frac{\chi^2(q)}{q}}$$

$$V \sim \chi^2(q) \Leftrightarrow V \sim \Gamma\left(\frac{q}{2}, 2\right)$$

$$E\left(V^{-k}\right) = \frac{1}{\Gamma(q/2)2^{\frac{q}{2}}} \int_0^\infty v^{\frac{q}{2}-k-1} e^{-\frac{v}{2}} dv = \frac{\Gamma(\frac{q}{2}-k)}{\Gamma(\frac{q}{2})2^k},$$

$$E[F] = \frac{q}{q-2}$$

$$Var[F] = \frac{2q^2(q+p-2)}{p(q-2)^2(q-4)}$$

Convergence concepts

Convergence in probability:

$$\{X_i\}_{i=1}^{\infty} \xrightarrow{P} X \text{ if } \forall\ \varepsilon > 0,\ \lim_{n\to\infty} P\left(\left|X_n - X\right| \geq \varepsilon\right) = 0.$$

Weak law of large numbers

$\{X_i\}_{i=1}^{\infty}$ iid, $\mathrm{E}[X_i] = \mu$ and $\mathrm{Var}(X_i) = \sigma^2 < \infty$. Then $\lim_{n\to\infty} P\left(\left|\bar{X}_n - \mu\right| < \varepsilon\right) = 1$